ALGEBRA II
FALL 2017

ASSIGNMENT 6.2
DUE OCTOBER 4

Let $R$ be a commutative ring. Given $a, b \in R$, we say that $d \in R$ is a *greatest common divisor* (GCD) of $a$ and $b$ provided:

(i) $d|a$ and $d|b$;

(ii) if $c \in R$, $c|a$ and $c|b$, then $c|d$.

**Exercise 1.** Show that if $R$ is a PID[1], then every pair of elements $a, b \in R$ has a GCD $d \in R$ and that the analogue of Bézout's Lemma holds: there exist $r, s \in R$ so that $ra + sb = d$. [*Suggestion:* Consider the ideal $\langle a, b \rangle$.]

**Exercise 2.** Prove that if $R$ is a domain, then GCDs are unique up to unit multiples.

**Exercise 3.** Show that the polynomial $x^4 + 4$ is reducible over $\mathbb{R}$ by factoring it.

[1]Recall that $R$ is called a *principal ideal domain* (PID) if it is a domain in which every ideal is principal, i.e. generated by a single element.