Algebra II
Fall 2017

Assignment 7.1
Due October 18

Exercise 1. Which of the following polynomials are irreducible over $\mathbb{Q}$?

a. $x^5 + 9x^4 + 12x^2 + 6$

b. $x^4 + x + 1$

c. $x^4 + x^2 + 2$

d. $x^5 + 5x^2 + 1$

e. $\frac{5}{2}x^5 + \frac{9}{2}x^4 + 15x^3 + \frac{3}{7}x^2 + 6x + \frac{3}{14}$

Exercise 2. Let p be a prime number. Determine the number of quadratic polynomials of the form $x^2 + ax + b$ that are irreducible over $\mathbb{F}_p$.

Exercise 3. In an integral domain, show that a and b are associates if and only if $\langle a \rangle = \langle b \rangle$.

Exercise 4. Show that 21 does not factor uniquely as a product of irreducibles in $\mathbb{Z}[\sqrt{-5}]$.

Exercise 5. In a ring R a *descending chain of ideals* is a sequence of ideals in R satisfying

$$I_1 \supseteq I_2 \supseteq I_3 \supseteq I_4 \supseteq \cdots .$$

We say that R satisfies the *descending chain condition* (DCC) if every descending chain of ideals in R stabilizes. A ring satisfying the DCC is called *Artinian.*

Show that a domain is Artinian if and only if it is a field.