ALGEBRA II
FALL 2017

ASSIGNMENT 7.2
DUE OCTOBER 18

Exercise 1. Suppose that $n \in \mathbb{Z}$ is not a perfect square. Let $\mathbb{Q}(\sqrt{n}) = \{a + b\sqrt{n} \mid a, b \in \mathbb{Q}\}$. Show that the function $N : \mathbb{Q}(\sqrt{n}) \to \mathbb{Q}$ given by $N(a + b\sqrt{n}) = a^2 - nb^2$ is multiplicative.

Exercise 2. Let R be a ring and M an R-module. Show that $r0_M = 0_M$ and $0_R m = 0_M$ for all $r \in R$ and $m \in M$.

Exercise 3. Let R be a commutative ring, M an R-module and $S \subseteq M$. Show that

$$\operatorname{Ann}(S) = \{r \in R \mid rm = 0 \text{ for all } m \in S\},$$

the *annihilator of* S, is an ideal in R.