Exercise 1. Let V be a vector space over F with subspaces U and W. Show that $U \cap W$ and $U + W = \{u + w \mid u \in U, w \in W\}$ are subspaces of V.

Exercise 2. Let V be a vector space over F with finite dimensional subspaces U and W. Prove that

$$\dim U + \dim W = \dim(U + W) + \dim U \cap W.$$

[*Suggestion:* Begin with a basis X of $U \cap W$ and complete it to bases Y and Z for U and W, respectively. Show that $X' = X \cup (Y \setminus X) \cup (Z \setminus X)$ is a basis for $U + W$.]

Exercise 3. Let $K/\mathbb{Q}$ be a field extension. Show that $\{\sqrt{2}, \sqrt{3}\} \subset K$ if and only if $\sqrt{2} + \sqrt{3} \in K$.