Algebra II
Fall 2017

Assignment 9.1
Due November 1

Exercise 1. Find a polynomial $f(x) \in \mathbb{Q}[x]$ so that $\mathbb{Q}\left(\sqrt{1+\sqrt{5}}\right)$ is isomorphic to $\mathbb{Q}[x]/\langle f \rangle$. Be sure to verify that f is irreducible over $\mathbb{Q}$.

Exercise 2. Let F be a field of characteristic p and let $f(x) = x^p - a \in F[x]$. Show that f is irreducible over F or splits in F. [*Suggestion:* Remember that $(a+b)^p = a^p + b^p$ in characteristic p.]

Exercise 3. Find $a, b, c \in \mathbb{Q}$ so that

$$\frac{1+\sqrt[3]{4}}{2-\sqrt[3]{2}} = a + b\sqrt[3]{2} + c\sqrt[3]{4}.$$

Exercise 4. Let K/F be fields and $S, T \subseteq K$. Prove that $F(S)(T) = F(S \cup T)$.