Algebra II
Fall 2017

Assignment 9.2
Due November 1

Exercise 1. Let p and q be distinct primes. Show that $x^2 - q$ is irreducible over $\mathbb{Q}(\sqrt{p})$.

Exercise 2. Determine the splitting field of $x^3 + 2x + 1$ over $\mathbb{F}_3 = \mathbb{Z}/3\mathbb{Z}$.

Exercise 3. Show that $f = x^3 + x + 1 \in \mathbb{Q}[x]$ is irreducible. Then according to Theorem 20.3, if α denotes any of its roots, we know that

$$\mathbb{Q}(\alpha) = \{a + b\alpha + c\alpha^2 \mid a, b, c \in \mathbb{Q}\}.$$

Therefore α^{-1}, α^{-2}, α^5 and α^{47} can all be expressed in the form $a + b\alpha + c\alpha^2$ with $a, b, c \in \mathbb{Q}$. Find a, b and c in each case.