



ALGEBRA II
FALL 2017

ASSIGNMENT 9.3
DUE NOVEMBER 1

Exercise 1. Let F be a field and $f \in F[x]$. Suppose K is a splitting field of f over F and $\alpha \in K$ is a root of f . Let $L = F(\alpha)$, so that we have $K/L/F$. Write $f = (x - \alpha)g$ in $L[x]$. Show that K is a splitting field of g over L .

Exercise 2. Let F be a field. Recall that two polynomials in $F[x]$ are called *relatively prime* if they have no non-constant factors in $F[x]$ in common (i.e. they have no common factor in $F[x]$ of positive degree). Prove that if $f, g \in F[x]$ are relatively prime, then they are relatively prime in $K[x]$ for any extension field K/F .

Exercise 3. Let F be a field, $f \in F[x]$ and $a, b \in F$ with $a \neq 0$. Show that $f(x)$ and $f(ax + b)$ have the same splitting field over F .