Name:_____

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Problem 1. Find the minimum value of

$$\frac{(x + 1/x)^6 - (x^6 + 1/x^6) - 2}{(x + 1/x)^3 + (x^3 + 1/x^3)}$$

for $x > 0$.

Problem 2. Evaluate $\sum_{n=0}^{\infty} \mathrm{Arccot}(n^2 + n + 1)$, where $\mathrm{Arccot}\, t$ for $t \geq 0$ denotes the number θ in the interval $0 < \theta \leq \pi/2$ with $\cot \theta = t$.

Problem 3. Prove or disprove: If x and y are real numbers with $y \geq 0$ and $y(y+1) \leq (x+1)^2$, then $y(y-1) \leq x^2$.

Problem 4. Find all real-valued continuously differentiable functions f on the real line such that for all x,

$$(f(x))^2 = \int_0^x \left[(f(t))^2 + (f'(t))^2\right] dt + 2018.$$

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