



PUTNAM SEMINAR  
FALL 2018

QUIZ 1  
DUE AUGUST 29

Name: \_\_\_\_\_

Start Time: \_\_\_\_\_

End Time: \_\_\_\_\_

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**Problem 1.** A function  $f$  is defined for all positive integers and satisfies

$$f(1) = 2018 \text{ and } f(1) + f(2) + \cdots + f(n) = n^2 f(n).$$

Compute the exact value of  $f(2018)$ .

**Problem 2.** Let  $Q_0(x) = 1$ ,  $Q_1(x) = x$ , and

$$Q_n(x) = \frac{(Q_{n-1}(x))^2 - 1}{Q_{n-2}(x)}$$

for all  $n \geq 2$ . Show that, whenever  $n$  is a positive integer,  $Q_n(x)$  is equal to a polynomial with integer coefficients.

**Problem 3.** Let  $A$  be the  $n \times n$  matrix whose entry in the  $i$ -th row and  $j$ -th column is

$$\frac{1}{\min(i, j)}$$

for  $1 \leq i, j \leq n$ . Compute  $\det(A)$ .

**Problem 4.** Show that every positive rational number can be written as a quotient of products of factorials of (not necessarily distinct) primes. For example,

$$\frac{10}{9} = \frac{2! \cdot 5!}{3! \cdot 3! \cdot 3!}.$$

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