Introduction to Abstract Mathematics — Assignment 11.2

Fall 2018

Exercise 1. Let $\equiv_{\mathbb{Z}} = \{(x, y) \in \mathbb{R}^2 \mid x - y \in \mathbb{Z}\}$. Prove that $\equiv_{\mathbb{Z}}$ is an equivalence relation on $\mathbb{R}$.

Exercise 2. Let A and B be sets, $f : A \to B$ a function, and $\equiv_f = \{(x, y) \in A^2 \mid f(x) = f(y)\}$. Prove that $\equiv_f$ is an equivalence relation on A.

Exercise 3. Let $\sim$ be a relation on a set A that is symmetric and transitive. What is wrong with the following "proof" that $\sim$ must automatically be reflexive as well?

Proof. If $a \sim b$, then by symmetry we also have $b \sim a$. Because $a \sim b$ and $b \sim a$, transitivity implies that $a \sim a$. Therefore $\sim$ is reflexive. □