Introduction to Abstract Mathematics — Assignment 12.1

Fall 2018

Exercise 1. Recall the equivalence relation on $A = \mathbb{R}^2 \setminus \{(0,0)\}$ defined by $P \sim Q$ if and only if there exists a nonzero $\lambda \in \mathbb{R}$ so that $P = \lambda Q$. Denote the equivalence class of (a,b) by $[a:b]$. Show that

$$A/\sim\, = \{[x:1] \,|\, x \in \mathbb{R}\} \cup \{[1:0]\}.$$

Exercise 2. Let A and $\sim$ be as in the previous exercise, and set $\mathbb{P}^1(\mathbb{R}) = \mathbb{R} \cup \{\infty\}$. Prove that the function $f : A/\sim\, \to \mathbb{P}^1(\mathbb{R})$ given by

$$f([a:b]) = \begin{cases} a/b & \text{if } b \neq 0, \\ \infty & \text{otherwise} \end{cases}$$

is a well-defined bijection.

Exercise 3. Let x be a nonzero real number. Prove that if $x + 1/x$ is an integer, then $x^n + 1/x^n$ is an integer for all $n \in \mathbb{N}$.