Introduction to Abstract Mathematics — Assignment 12.3
Fall 2018

**Exercise 1.** Define a sequence $\{a_n\}$ by setting $a_0 = 2$, $a_1 = 3$, $a_2 = 13$, and $a_{n+1} = 3a_n - 4a_{n-2}$ for $n \geq 2$. Prove that

$$a_n = (n+1)2^n + (-1)^n.$$

**Exercise 2.** Call a positive integer $n$ a *McNugget number* if there exist $a_1, a_2, a_3 \in \mathbb{N}_0$ so that $n = 6a_1 + 9a_2 + 20a_3$. Prove that 43 is the smallest non-McNugget number.

**Exercise 3.** In the game of Nim, two players take turns removing matches from two piles. At each turn, a player removes some (non-zero) number of matches from *one* of the piles. The player who removes the last match wins. Prove that if the piles of matches initially have the same size $n \in \mathbb{N}$, then the second player can always win.