Introduction to Abstract Mathematics — Assignment 2.1
Fall 2018 — Due September 5

Exercise 1. Let $\otimes$ denote the *exclusive or* connective, so that $P \otimes Q$ means "P or Q, but not both."

a. Make a truth table for $P \otimes Q$.

b. Find a formula using only the connectives $\wedge$, $\vee$, and $\neg$ that is equivalent to $P \otimes Q$. Justify your answer with a truth table.

Exercise 2. Verify the following logical equivalences.

a. $P \cong P \wedge P \cong P \vee P$

b. If C is a contradiction, then $P \wedge C$ is a contradiction, while $P \vee C \cong P$.

c. If T is a tautology, then $P \wedge T \cong P$, while $P \vee T$ is a tautology.

Exercise 3. Use established logical equivalences to verify the following.

a. $(P \Rightarrow R) \wedge (Q \Rightarrow R) \cong (P \vee Q) \Rightarrow R$

b. $(P \Rightarrow R) \vee (Q \Rightarrow R) \cong (P \wedge Q) \Rightarrow R$

c. $(P \Rightarrow Q) \wedge (Q \Rightarrow R) \cong (P \Rightarrow R) \wedge [(P \Leftrightarrow Q) \vee (R \Leftrightarrow Q)]$

d. $(P \Rightarrow Q) \vee (Q \Rightarrow R)$ is a tautology.

Exercise 4. Find a formula involving only the connectives $\wedge$, $\vee$, and $\neg$ that has the following truth table:

P	Q	???
T	T	T
T	F	T
F	T	F
F	F	T