Introduction to Abstract Mathematics
Fall 2018

Assignment 2.3
Due September 5

Exercise 1. Negate the following statements and then reexpress the results as equivalent positive statements.

a. Everyone has a roommate who dislikes everyone.

b. There is someone in the first-year class who doesn't have a roommate.

c. Everyone likes someone, but no one likes everyone.

d. $(\exists x \in \mathbb{R})(\forall y \in \mathbb{R})[y > x \Rightarrow (\exists z \in \mathbb{R})(z^2 + 5z = y)]$

e. $(\forall a \in A)(\exists b \in B)(a \in C \Leftrightarrow b \in C)$

Exercise 2. Negate the uniqueness quantifier

$$(\exists! x \in S)(P(x)) = (\exists x \in S)[P(x) \wedge (\forall y \in S)(x \neq y \Rightarrow \neg P(y))]$$

and reexpress the negation as sentence in English.

Exercise 3. Use the preceding exercise to negate the statement

$$P = (\exists! x \in \mathbb{N})((x-4)^2 = 9).$$

Which statement is true, P or $\neg P$? What if 9 is replaced by 25?

Exercise 4.

a. Show that $(\exists x \in S)(P(x) \vee Q(x)) \cong (\exists x \in S)(P(x)) \vee (\exists x \in S)(Q(x))$.

b. Use part **a** to show that $(\exists x \in S)(P(x) \Rightarrow Q(x)) \cong (\forall x \in S)(P(x)) \Rightarrow (\exists x \in S)(Q(x))$.