Introduction to Abstract Mathematics — Assignment 3.1
Fall 2018 — Due September 12

Exercise 1. Prove that if $n \in \mathbb{Z}$ is even, then n^2 is even.

Exercise 2. Let $a, b \in \mathbb{R}$.

a. Prove that if $0 < a < b$, then $a^2 < b^2$.

b. Prove that if $a < b$, then $a < \frac{a+b}{2} < b$.

Exercise 3. Consider the following theorem.

Theorem 1. *Suppose* x *is a real number and* $x \neq 4$. *If* $\frac{2x-5}{x-4} = 3$, *then* $x = 7$.

a. What's wrong with the following proof of the theorem?

Proof. Suppose $x = 7$. Then $\frac{2x-5}{x-4} = \frac{2(7)-5}{7-4} = \frac{9}{3} = 3$. Therefore if $\frac{2x-5}{x-4} = 3$, then $x = 7$. □

b. Give a correct proof of the theorem.

c. What statement does the proof in part **a** actually prove?

Exercise 4. Show that an implication is logically equivalent to its contrapositive, but not equivalent to its converse.