



INTRODUCTION TO ABSTRACT MATHEMATICS
FALL 2018

ASSIGNMENT 4.1
DUE SEPTEMBER 19

Exercise 1. Let $a, b \in \mathbb{R}^+$.

- a. Prove that $a < b$ if and only if $a^2 < b^2$.
- b. Prove that $a < b$ if and only if $\sqrt{a} < \sqrt{b}$.
- c. If $a \neq b$, prove the *Arithmetic Mean-Geometric Mean Inequality*

$$\sqrt{ab} < \frac{a+b}{2}.$$

Exercise 2. Let $n \in \mathbb{Z}$. Prove that exactly one of n , $n + 1$ is even.

Exercise 3. Suppose that $x, y \in \mathbb{R}$. Prove that if $x^2y = 2x + y$, then $y \neq 0$ implies $x \neq 0$.
[*Suggestion:* First show that $P \Rightarrow (Q \Rightarrow R) \cong (P \wedge Q) \Rightarrow R$.]

Exercise 4. Suppose that $x, y \in \mathbb{R}$, that $x + y = 2y - x$, and that x and y are not both zero. Prove that $y \neq 0$.