Introduction to Abstract Mathematics
Fall 2018

Assignment 4.3
Due September 19

Exercise 1. Prove that $(x-1)^2 \leq x+1$ for all real numbers $x \in [0,3]$.

Exercise 2. Prove that $n! \geq 2^n$ for every integer $n \geq 4$.

Exercise 3. A real-valued function f of one variable is called *convex* if

$$f(tx + (1-t)y) \leq tf(x) + (1-t)f(y)$$

for all real numbers x, y and all t with $0 \leq t \leq 1$. Prove that $f(x) = x^2$ is convex.

Exercise 4. Let $f(n) = n^2 + n + 41$.

a. Show that $f(1), f(2), \ldots, f(10)$ are prime.

b. Prove or disprove: $f(n)$ is prime for all $n \in \mathbb{N}$.