Introduction to Abstract Mathematics — Assignment 5.1

Fall 2018 — Due September 26

Exercise 1. Let $f = x^4 - 2x^2 + 5$.

a. Find the four roots of f.

b. Use part **a** to write f as the product of two quadratic polynomials with real coefficients.

Exercise 2. Prove that there exists a real number y so that the equation $z^2 + 5z = x$ has a (real) solution for all (real) $x > y$.

Exercise 3. Let $n \in \mathbb{Z}$.

a. Prove that $n(n+1)(2n+1)$ is divisible by 6. [*Suggestion:* Consider the possible remainders when n is divided by 6.]

b. If n is odd, prove that $n^2 - 1$ is divisible by 8. [*Suggestion:* Consider the possible remainders when n is divided by 8.]

c. Prove that if n is simultaneously a square and a cube (e.g. $64 = 8^2 = 4^3$), then either n or $n-1$ is divisible by 7. [*Suggestion:* Consider the possible remainders after division by 7.]

Exercise 4. Let $n \in \mathbb{N}$, $n \geq 2$. Given $a \in \mathbb{Z}$, let $r(a)$ denote the remainder when a is divided by n. Prove that $r(a \cdot r(bc)) = r(r(ab) \cdot c)$ for all $a, b, c \in \mathbb{Z}$. [*Suggestion:* Compute the remainder when abc is divided by n in two different ways.]