INTRODUCTION TO ABSTRACT MATHEMATICS — ASSIGNMENT 5.1

FALL 2018 — DUE SEPTEMBER 26

Exercise 1. Prove that $\{2a + 3b \mid a, b \in \mathbb{N}_0\} = \{n \in \mathbb{N}_0 \mid n \neq 1\}$. [*Suggestion:* To prove $\supset$, first show that 0, 2, 3, 4 all have the form $2a + 3b$. Then consider what happens as you add multiples of 3 to these.]

Exercise 2. Prove that $\{x \in S \mid P(x)\} = \{x \in S \mid Q(x)\}$ if and only if $(\forall x \in S)(P(x) \Leftrightarrow Q(x))$ is true.

Exercise 3. Let S be a set and suppose that to each $x \in S$ we assign a subset $Y(x) \subset S$. Let

$$Z = \{x \in S \mid x \notin Y(x)\}.$$

Show that $Z \neq Y(x)$ for any $x \in S$. [*Suggestion:* Argue by contradiction. If $Z = Y(x)$ for some $x \in S$, then $x \in Z$ or $x \notin Z$. Use the membership criterion for Z to show that both situations are untenable.]