Introduction to Abstract Mathematics — Assignment 5.3
Fall 2018 — Due September 26

Exercise 1. Let A, B and C be sets. Prove the following identities.

a. $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ and $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

b. $(A \setminus B) \cap C = (A \cap C) \setminus B$ and $(A \setminus B) \cup C = (A \cup C) \setminus (B \setminus C)$

c. $(A \setminus B) \setminus C = (A \setminus C) \setminus B$ and $A \setminus (B \setminus C) = (A \setminus B) \cup (A \cap C)$.

Exercise 2. The *symmetric difference* of two sets A and B is defined to be

$$A \Delta B = (A \setminus B) \cup (B \setminus A).$$

Verify the following properties of the symmetric difference.

a. $A \Delta B = B \Delta A$.

b. $A \Delta B = (A \cup B) \setminus (A \cap B)$.

c. $A \Delta B = A \cup B$ if and only if $A \cap B = \varnothing$.

Exercise 3. Let A, B, C be sets.

a. Show that

$$(A \Delta B) \Delta C = [(A \cup B \cup C) \setminus ((A \cap B) \cup (A \cap C) \cup (B \cap C))] \cup (A \cap B \cap C).$$

b. Use part **a** to conclude that $(A \Delta B) \Delta C = (B \Delta C) \Delta A$.

c. Use the commutativity of Δ and part **b** to show that $(A \Delta B) \Delta C = A \Delta (B \Delta C)$.