



INTRODUCTION TO ABSTRACT MATHEMATICS  
FALL 2018

ASSIGNMENT 6.1  
DUE OCTOBER 3

**Exercise 1.** Let  $a < b$  be real numbers. Show that  $(a, b)$  is equal to a union of closed intervals, but that  $[a, b]$  is not equal to a union of open intervals.

**Exercise 2.** As in class, given  $n \geq 2$  and  $0 \leq r < n$ , define

$$R_n(r) = \{a \in \mathbb{Z} \mid (\exists q \in \mathbb{Z})(a = nq + r)\} = \{nq + r \mid q \in \mathbb{Z}\}.$$

- a. Find, with proof,  $R_3(1) \cap R_2(0)$ .
- b. Find, with proof,  $R_5(2) \cap R_6(5)$ .
- c. Make a general conjecture about the structure of  $R_m(a) \cap R_n(b)$  when  $\gcd(m, n) = 1$ .

**Exercise 3.** Given a set  $X$  let  $\tau = \{U \subset X \mid X \setminus U \text{ is finite}\} \cup \{\emptyset\}$ . Show that  $\tau$  is a topology on  $X$ .