Introduction to Abstract Mathematics — Fall 2018

Assignment 8.1 — Due October 24

Exercise 1. Let A and B be sets. Prove that $A \subset B$ if and only if $\mathcal{P}(A) \subset \mathcal{P}(B)$.

Exercise 2. Let S be a set and $\mathcal{F} \subset \mathcal{P}(S)$. Prove the following.

a. For any $X \in \mathcal{P}(S)$, $X \subset \bigcap_{A \in \mathcal{F}} A$ if and only $X \subset A$ for all $A \in \mathcal{F}$. [*Suggestions:* Use a result we proved in class to prove the forward implication. Use another result from class and negation to prove the converse.]

b. Use part **a** to prove that

$$\bigcap_{A \in \mathcal{F}} \mathcal{P}(A) = \mathcal{P}\left(\bigcap_{A \in \mathcal{F}} A\right).$$

Exercise 3. Let S be a set and $\mathcal{F} \subset \mathcal{P}(S)$.

a. Prove that

$$\bigcup_{A \in \mathcal{F}} \mathcal{P}(A) \subset \mathcal{P}\left(\bigcup_{A \in \mathcal{F}} A\right).$$

Note that this implies $A' \subset \bigcup_{A \in \mathcal{F}} A$ for any $A' \in \mathcal{F}$ (why?).

b. Show that equality occurs in part **a** if and only if there is a $A' \in \mathcal{F}$ so that $\mathcal{F} \subset \mathcal{P}(A')$. [*Suggestion:* Note that if we have equality, then $\bigcup_{A \in \mathcal{F}} A \in \bigcup_{A \in \mathcal{F}} \mathcal{P}(A)$.]