Introduction to Abstract Mathematics — Assignment 8.2
Fall 2018 — Due October 26

Exercise 1. Let $f : X \to Y$ be a function. Given $x \in X$, recall that we defined $f(x)$ to be the unique element of Y so that $(x, f(x)) \in f$ (so that $(x, y) \in f$ if and only if $y = f(x)$). If $g : X \to Y$ is also a function, prove that $f = g$ (which is an equality of sets) if and only if $f(x) = g(x)$ for all $x \in X$ (which is how we usually think of function equality).

Exercise 2. Let $f = \{(x, n) \in \mathbb{R} \times \mathbb{Z} \,|\, n \leq x < n + 1\}$. Explain why f is a function from $\mathbb{R}$ to $\mathbb{Z}$. What is $f(\pi)$? What is $f(-\pi)$? Is f one-to-one? Onto?

Exercise 3. Define $f : \mathbb{R} \setminus \{-1\} \to \mathbb{R}$ by $f(x) = 2x/(x+1)$. Prove that f is one-to-one but not onto. Is it possible to modify the codomain so that the same formula yields a bijection? Prove your answer.

Exercise 4. Let $A = \mathbb{R} \setminus \{1\}$ and define $f : A \to A$ by $f(x) = (x + 1)/(x - 1)$. Prove that f is a bijection.

Exercise 5. Let S be a set, $Y = \mathcal{P}(S)$ and $X = \mathcal{P}(Y)$. Define $f : X \to Y$ by

$$f(\mathcal{F}) = \bigcup_{A \in \mathcal{F}} A.$$

Is f one-to-one? Onto?