Introduction to Abstract Mathematics — Assignment 9.1
Fall 2018 — Due October 31

Exercise 1. Let $a, b \in \mathbb{R}$. Suppose that $b > 0$ and that $a \neq 0$. Prove that the polynomial $x^2 + ax - b$ has two distinct real roots and that exactly one of them lies in the interval $(-\sqrt{b}, \sqrt{b})$. [*Suggestion:* First show that if α and β are the roots of $x^2 + ax - b$, then $\alpha + \beta = -a$ and $\alpha\beta = -b$.]

Exercise 2. Let $f : X \to Y$, $g : Y \to Z$ and $h : Z \to W$ be functions. Prove that function composition is associative, i.e. that $h \circ (g \circ f) = (h \circ g) \circ f$. [*Suggestion:* Use the fact, proven in an earlier assignment, that $f = g$ if and only if $f(x) = g(x)$ for all $x \in X$.]

Exercise 3. Let S be a set. Prove that if $f : S \to \mathcal{P}(S)$ is any function, then f is not surjective. [*Suggestion:* See Exercise 3 in Assignment 5.2.]