Introduction to Abstract Mathematics — Assignment 9.2
Fall 2018 — Due October 31

Exercise 1. Let $f : X \to Y$ and $g : Y \to Z$ be functions. Prove the following.

a. If f and g are injective, then so is $g \circ f$.

b. If f and g are surjective, then so is $g \circ f$.

c. If f and g are bijective, then so is $g \circ f$.

Exercise 2. We have seen that if $f : X \to Y$ is a bijection, then $f^{-1} = \{(f(x), x) \mid x \in X\}$ is also a function from Y to X. Prove that $f \circ f^{-1} = 1_Y$ and $f^{-1} \circ f = 1_X$.

Exercise 3. Let $f : X \to Y$ be a function. Prove that f is injective if and only if there is a function $g : Y \to X$ so that $g \circ f = 1_X$.

Exercise 4. Recall that a set S is *finite* if there exists an $n \in \mathbb{N}$ and a bijection $f : I_n \to S$ (where $I_n = \{i \in \mathbb{N} \mid i \leq n\}$).

a. We say S is *infinite* if it is not finite. Negate the definition of *finite* to obtain a positive definition of *infinite*.

b. If $n \in \mathbb{N}$ and $f : I_n \to \mathbb{N}$ is any function, show that $N = 1 + \sum_{i=1}^{n} f(i)$ is a natural number not in the image of f.

c. Use parts **a** and **b** to prove that $\mathbb{N}$ is infinite.