Putnam Seminar
Fall 2019

Quiz 2
Due September 11

Name:____________________________

Start Time:________________________

End Time:_________________________

Problem 1. Let S be a subset of $\{1, 2, \ldots, 2n\}$ with $n+1$ elements. Show that one can choose distinct elements $a, b \in S$ such that a divides b.

Problem 2. For a partition π of $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, let $\pi(x)$ be the number of elements in the part containing x. Prove that for any two partitions π and π', there are two distinct numbers x and y in $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ such that $\pi(x) = \pi(y)$ and $\pi'(x) = \pi'(y)$.

Problem 3. Let a_j, b_j, c_j be integers for $1 \leq j \leq N$. Assume for each j, at least one of a_j, b_j, c_j is odd. Show that there exist integers r, s, t such that $ra_j + sb_j + tc_j$ is odd for at least $4N/7$ values of j, $1 \leq j \leq N$.

Problem 4. Prove that, for every set $X = \{x_1, x_2, \ldots, x_n\}$ of real numbers, there exists a non-empty subset S of X and an integer m such that

$$\left| m + \sum_{s \in S} s \right| \leq \frac{1}{n+1}$$

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