MODERN ALGEBRA II — ASSIGNMENT 1.1

FALL 2019 — DUE SEPTEMBER 4

Exercise 1. A ring R such that $a^2 = a$ for all $a \in R$ is called *Boolean*. Prove that every Boolean ring R is commutative and that $2a = 0$ for all $a \in R$.

Exercise 2. Let R be a ring, S a set, and

$$R^S = \{f : S \to R\},$$

the set of all functions from S to R. Prove that R^S is a ring under point-wise operations.

Exercise 3. If $A = \mathbb{Z} \oplus \mathbb{Z}$, prove that $\operatorname{End} A$ is not commutative. Can you identify $\operatorname{End} A$?

Exercise 4. Given a ring R, let

$$Z(R) = \{a \in R \mid ab = ba \text{ for all } b \in R\}.$$

Prove that $Z(R)$ is a subring of R, called the *center* of R.

Exercise 5. Let $\alpha \in \mathbb{C}$. Prove that if α is a root of a monic quadratic polynomial with integer coefficients, then

$$\mathbb{Z}[\alpha] = \{a + b\alpha \mid a, b \in \mathbb{Z}\}$$

is a subring of $\mathbb{C}$.