Modern Algebra II
Fall 2019

Assignment 1.2
Due September 4

Exercise 1. Let $p \in \mathbb{N}$ be a prime and let $\mathbb{Z}_{(p)}$ denote the set of all rational numbers with denominators *not* divisible by p.

- **a.** Show that $\mathbb{Z}_{(p)}$ is a subring of $\mathbb{Q}$.
- **b.** Determine $\mathbb{Z}_{(p)}^{\times}$.

Exercise 2. Let $\mathbb{H}$ be the set of all 2×2 matrices over $\mathbb{C}$ of the form

$$\begin{pmatrix} z & w \\ -\overline{w} & \overline{z} \end{pmatrix},$$

where the bar denotes complex conjugation. Prove that $\mathbb{H}$ is a noncommutative division ring.[1]

Exercise 3. Let R be a ring and $a, b, c \in R$. Prove the following *left cancellation law*: if $a \neq 0$ and a is not a zero divisor, then $ab = ac$ implies $b = c$.

Exercise 4. Prove that if F is a field, then every nonzero element of $\mathrm{M}_n(F)$ is either a unit or a zero divisor. Show that this is no longer true if F is replaced with $\mathbb{Z}$.

[1] A *division ring* (or *skew field*) is a ring (not necessarily commutative) in which every nonzero element is a unit.