Modern Algebra II
Fall 2019

Assignment 1.3
Due September 4

Exercise 1. Prove that a ring with a left cancellation law has no zero divisors. In particular, a commutative, left cancellative ring is a domain.

Exercise 2. Let R be a ring with no zero divisors. Prove that for all $n \geq 2$ and all $a_1, a_2, \dots, a_n \in R$,

$$a_1 a_2 \cdots a_n = 0 \;\; \Rightarrow \;\; a_i = 0 \text{ for some } i.$$

Exercise 3. Let S be a set, R be a ring and $a \in S$. Prove that the *evaluation map* $E_a : R^S \to R$, given by

$$f \mapsto f(a),$$

is a homomorphism of rings. What is $\ker E_a$?

Exercise 4. Let R and R' be rings and $f : R \to R'$ a function satisfying $f(a+b) = f(a)+f(b)$ and $f(ab) = f(a)f(b)$ for all $a, b \in R$. In class I claimed that these conditions do *not* imply that $f(1_R) = 1_{R'}$ (which is why we included this additional equality in our definition of ring homomorphism). Verify my claim. That is, give an example of R, R' and a nonzero f for which $f(1_R) \neq 1_{R'}$.