Modern Algebra II
Fall 2019

Assignment 10.3
Due November 20

Exercise 1. Let K/F be fields and suppose $\alpha \in K$ is algebraic over F with minimal polynomial $p \in F[X]$. If $q \in F[X]$ is monic, irreducible and satisfies $q(\alpha) = 0$, show that $p = q$.

Exercise 2. Let F be a field and let $f, g \in F[X]$ be nonzero. Suppose that K is an extension of F and there is an $\alpha \in K$ so that $f(\alpha) = g(\alpha) = 0$. Prove that f and g have a nontrivial common factor in $F[X]$.[1] Conclude that two polynomials over F have a nontrivial common factor over F if and only if they share a root in an extension of F.

Exercise 3. Let α be a root of $X^3 + X^2 + 1$ in some extension of $\mathbb{F}_2 = \mathbb{Z}/2\mathbb{Z}$. What is the degree of the minimal polynomial of α over $\mathbb{F}_2$?

Exercise 4. Lang, Exercise VII.1.23.

[1] The polynomial $X - \alpha$ doesn't count as a common factor: it divides both f and g, but not necessarily over F.