MODERN ALGEBRA II — ASSIGNMENT 11.1
FALL 2019 — DUE DECEMBER 2

Exercise 1. Let F be a field and suppose $f \in F[X]$ has positive degree n. Use the division algorithm to show that the cosets of 1, X, $X^2, \ldots, X^{n-1}$ in $F[X]/(f)$ form an F-basis for $F[X]/(f)$.

Exercise 2. Let K/F be fields and suppose $t \in K$ is transcendental over F. Show that the evaluation map $E_t : F[X] \to K$ yields an isomorphism $F(X) \cong F(t)$.

Exercise 3. Let F be a field and let $a \in F$. If $a \neq \square$ in F, prove that $[F(\sqrt{a}) : F] = 2$.

Exercise 4. Let p, q be distinct primes. Find the minimal polynomial of $\sqrt{p} + \sqrt{q}$ over $\mathbb{Q}$. [*Suggestion:* Use Exercises 3 and 10.1.3c.]