



MODERN ALGEBRA II
FALL 2019

ASSIGNMENT 11.2
DUE DECEMBER 2

Exercise 1. Let $K/k/F$ be a tower of fields. If $\alpha \in K$ is algebraic over F , show that

$$[k(\alpha) : k] \leq [F(\alpha) : F].$$

Exercise 2. Let K/F be an extension of fields and suppose $\alpha_1, \dots, \alpha_n \in K$ are algebraic over F . Prove that

$$[F(\alpha_1, \dots, \alpha_n) : F] \leq \prod_{i=1}^n [F(\alpha_i) : F].$$

In particular, $F(\alpha_1, \dots, \alpha_n)$ is a finite extension of F . [*Suggestion:* Induct on n .]

Exercise 3. Let K/F be fields and let $\alpha \in K$. Prove that if α is algebraic of odd degree over F , then so is α^2 and $F(\alpha) = F(\alpha^2)$.