Exercise 1. Let $p, q \in \mathbb{Z}$ and set

$$R = \left\{ \begin{pmatrix} a & -bq \\ b & a - bp \end{pmatrix} \middle| a, b \in \mathbb{Z} \right\}.$$

Prove that R is a subring of $M_2(\mathbb{Z})$ isomorphic to $\mathbb{Z}[\alpha]$, where $\alpha \in \mathbb{C}$ is a root of $x^2 + px + q$ (see Exercise 1.1.5).

Exercise 2. Let $\varphi : R \to R'$ be a homomorphism of rings. Prove that if char $R \neq 0$, then char R' divides char R.

Exercise 3. Lang: Exercise III.3.24.