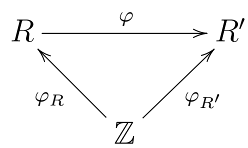
Modern Algebra II
Fall 2019

Assignment 2.2
Due September 11

Exercise 1. Let R be a ring and let $\varphi_R : \mathbb{Z} \to R$ denote the map $n \mapsto n \cdot 1_R$.

a. Prove that φ_R is the *only* homomorphism from $\mathbb{Z}$ to R.

b. Let $\varphi : R \to R'$ be a homomorphism of rings. Prove that the following diagram commutes:



Exercise 2. Let R be a commutative ring with prime characteristic p. Prove that for all $a, b \in R$ and all $n \in \mathbb{N}$ one has $(a+b)^{p^n} = a^{p^n} + b^{p^n}$.

Exercise 3. An element a in a ring R is called *nilpotent* if there is an $n \in \mathbb{N}$ so that $a^n = 0$.

a. Give an example of a ring with nonzero nilpotents.

b. If $a, b \in R$ are nilpotent and $ab = ba$, prove that $a + b$ is nilpotent.

Exercise 4. Let R be a ring. Prove that if $a \in R$ is nilpotent, then $1 \pm a \in R^{\times}$.