Modern Algebra II
Fall 2019

Assignment 7.1
Due October 30

Exercise 1.

a. Show that $X+1$ is a unit in $\mathbb{Z}[[X]]$, but is not a unit in $\mathbb{Z}[X]$.

b. Show that X^2+3X+2 is irreducible in $\mathbb{Z}[[X]]$, but is not irreducible in $\mathbb{Z}[X]$.

Exercise 2. Let R be a commutative ring and $S \subset R$ a proper multiplicative subset.

a. Prove that for all $s \in S$ and $a \in R$, if $sa = 0$, then $a = 0$.

b. Show that for all $\frac{a}{s}, \frac{b}{t} \in S^{-1}R$ one has $\frac{a}{s} = \frac{b}{t}$ if and only if $at - bs = 0$.

Exercise 3. Let S be a multiplicative set in a commutative ring R. Suppose $s \in S$, $a \in R$ and $a \neq 0$. Prove that if $as = 0$, then $\frac{a}{1} = \frac{0}{1}$. Conclude that if $0 \in S$, then $S^{-1}R = \left\{\frac{0}{1}\right\}$.

Exercise 4. If $S \subset \mathbb{Z}$ is a proper multiplicative set, then we can view $S^{-1}\mathbb{Z}$ as a subset of $\mathbb{Q}$.

a. Show that if S is the set of even integers, then $S^{-1}\mathbb{Z} = \mathbb{Q}$.

b. State and prove a condition on (a general) S that guarantees $S^{-1}\mathbb{Z} = \mathbb{Q}$.