Modern Algebra II
Fall 2019

Assignment 9.2
Due November 13

Exercise 1. Let F be a field and define

$$F[X;\mathbb{Q}_0^+] = \left\{ \sum_{r\in\mathbb{Q}_0^+} a_r X^r \,\middle|\, a_r \in F, \text{ almost all } a_r = 0 \right\},$$

where "almost all" means all but finitely many.

a. Let $r \in \mathbb{Q}^+$. Show that the rule $f(X) \mapsto f(X^r)$ defines an automorphism E_r of $F[X;\mathbb{Q}_0^+]$.

b. Given $f_1, \ldots, f_n \in F[X;\mathbb{Q}_0^+]$, show that there is an $m \in \mathbb{N}$ so that $E_m(f_i) \in F[X]$ for all i.

c. Show that $F[X;\mathbb{Q}_0^+]$ is a GCD domain.

Exercise 2.

Fix $r_0 \in \mathbb{Q}^+$ and let $S = \{X^r \mid r > r_0\} \subset F[X;\mathbb{Q}_0^+]$.

a. Show that (S) cannot be finitely generated.

b. Find (with proof) $\gcd S$.