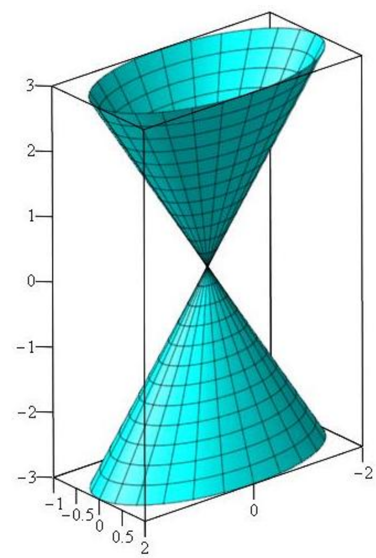
Equation:

$$\frac{z^2}{c^2} = \frac{x^2}{a^2} + \frac{y^2}{b^2}$$

Graph:



Cross sections:

$x = k:\quad \dfrac{z^2}{(ck/a)^2} - \dfrac{y^2}{(bk/a)^2} = 1$ — Hyperbolas whose branches open along the z-axis, collapsing to a pair of lines through the origin when $k = 0$.

$y = k:\quad \dfrac{z^2}{(ck/b)^2} - \dfrac{x^2}{(ak/b)^2} = 1$ — Hyperbolas whose branches open along the z-axis, collapsing to a pair of lines through the origin when $k = 0$.

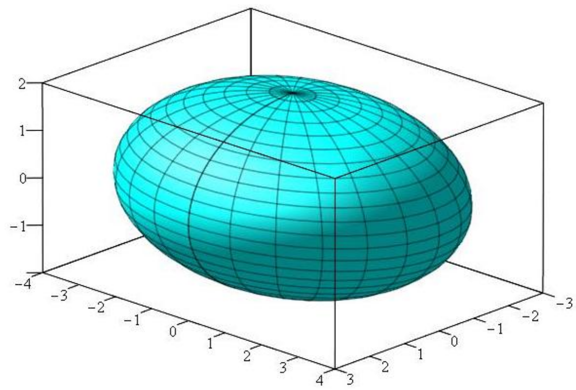
$z = k:\quad \dfrac{x^2}{(ak/c)^2} + \dfrac{y^2}{(bk/c)^2} = 1$ — Ellipses whose dimensions increase linearly with $|k|$, collapsing to a point when $k = 0$.

Ellipsoids

Equation:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

Graph:



Cross sections:

$x = k:$ $\quad \dfrac{y^2}{(\frac{b}{a}\sqrt{a^2 - k^2})^2} + \dfrac{z^2}{(\frac{c}{a}\sqrt{a^2 - k^2})^2} = 1$

Ellipses whose dimensions decrease as $|k| \to a^-$, collapsing to the origin when $|k| = a$. Cross sections with $|k| > a$ are empty.

$y = k:$ $\quad \dfrac{x^2}{(\frac{a}{b}\sqrt{b^2 - k^2})^2} + \dfrac{z^2}{(\frac{c}{b}\sqrt{b^2 - k^2})^2} = 1$

Ellipses whose dimensions decrease as $|k| \to b^-$, collapsing to the origin when $|k| = b$. Cross sections with $|k| > b$ are empty.

$z = k:$ $\quad \dfrac{x^2}{(\frac{a}{c}\sqrt{c^2 - k^2})^2} + \dfrac{y^2}{(\frac{b}{c}\sqrt{c^2 - k^2})^2} = 1$

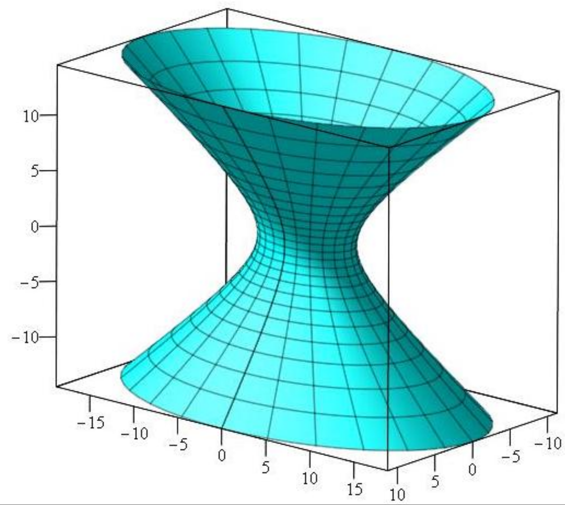
Ellipses whose dimensions decrease as $|k| \to c^-$, collapsing to the origin when $|k| = c$. Cross sections with $|k| > c$ are empty.

One-sheeted hyperboloids

Equation:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$$

Graph:



Cross sections:

$x = k$: $$\frac{y^2}{(\frac{b}{a}\sqrt{|a^2 - k^2|})^2} - \frac{z^2}{(\frac{c}{a}\sqrt{|a^2 - k^2|})^2} = \pm 1$$

Hyperbolas. Sign matches $a^2 - k^2$. For $|k| < a$, they open along y-axis. For $|k| > a$, they open along z-axis. When $|k| = a$, hyperbolas collapse to a pair of lines through the origin.

$y = k$: $$\frac{x^2}{(\frac{a}{b}\sqrt{|b^2 - k^2|})^2} - \frac{z^2}{(\frac{c}{b}\sqrt{|b^2 - k^2|})^2} = \pm 1$$

Hyperbolas. Sign matches $b^2 - k^2$. For $|k| < b$, they open along x-axis. For $|k| > b$, they open along z-axis. When $|k| = b$, hyperbolas collapse to a pair of lines through the origin.

$z = k$: $$\frac{x^2}{(\frac{a}{c}\sqrt{c^2 + k^2})^2} + \frac{y^2}{(\frac{b}{c}\sqrt{c^2 + k^2})^2} = 1$$

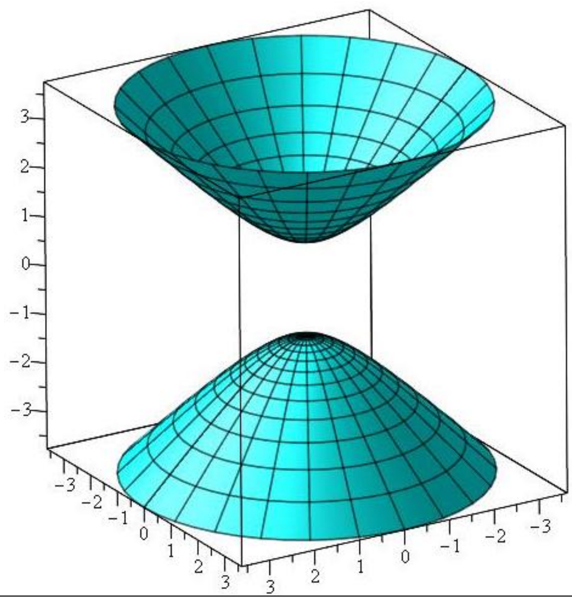
Ellipses whose dimensions increase roughly linearly as $|k|$ increases, achieving their minimum (positive) size when $k = 0$.

Two-sheeted hyperbolids

Equation:

$$-\frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

Graph:



Cross sections:

$$x = k: \quad \frac{-y^2}{(\frac{b}{a}\sqrt{a^2+k^2})^2} + \frac{z^2}{(\frac{c}{a}\sqrt{a^2+k^2})^2} = 1$$

Hyperbolas opening along the z-axis whose z-intercepts increase roughly linearly with $|k|$.

$$y = k: \quad \frac{-x^2}{(\frac{a}{b}\sqrt{b^2+k^2})^2} + \frac{z^2}{(\frac{c}{b}\sqrt{b^2+k^2})^2} = 1$$

Hyperbolas opening along the z-axis whose z-intercepts increase roughly linearly with $|k|$.

$$z = k: \quad \frac{x^2}{(\frac{a}{c}\sqrt{k^2-c^2})^2} + \frac{y^2}{(\frac{b}{c}\sqrt{k^2-c^2})^2} = 1$$

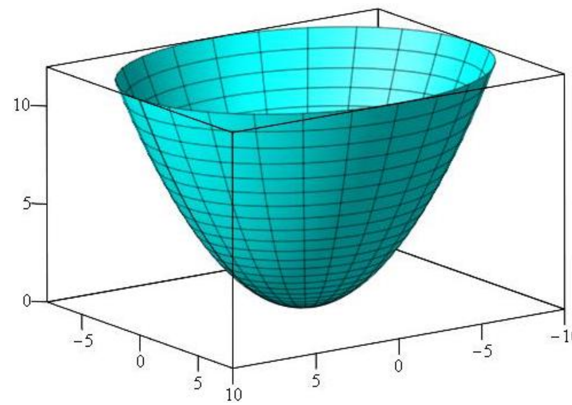
Ellipses whose dimensions increase roughly linearly with $|k|$, collapsing to points when $|k| = c$. Cross sections with $|k| < c$ are empty.

Paraboloids

Equation:

$$\frac{z}{c} = \frac{x^2}{a^2} + \frac{y^2}{b^2}$$

Graph:



Cross sections:

$x = k:$ $$z = \frac{cy^2}{b^2} + \frac{ck^2}{a^2}$$

Parabolas opening along the z-axis: upward if $c > 0$, downward if $c < 0$. z-intercept increases quadratically in k.

$y = k:$ $$z = \frac{cx^2}{a^2} + \frac{ck^2}{b^2}$$

Parabolas opening along the z-axis: upward if $c > 0$, downward if $c < 0$. z-intercept increases quadratically in k.

$z = k:$ $$\frac{x^2}{\left(a\sqrt{k/c}\right)^2} + \frac{y^2}{\left(b\sqrt{k/c}\right)^2} = 1$$

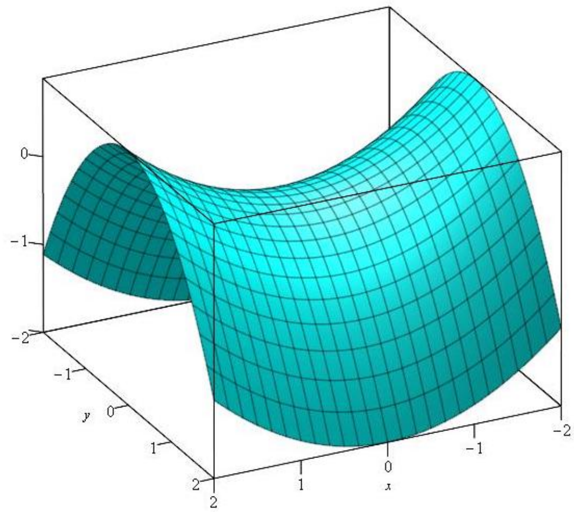
Ellipses whose dimensions are proportional to $\sqrt{|k|}$, collapsing to the origin when $k = 0$. Cross sections are empty unless k and c have the same sign.

Hyperbolic Paraboloids

Equation:

$$\frac{z}{c} = \frac{x^2}{a^2} - \frac{y^2}{b^2}$$

Graph:



Cross sections:

$x = k:$ $\quad z = \frac{-cy^2}{b^2} + \frac{ck^2}{a^2}$

Parabolas opening along the z-axis: downward if $c > 0$, upward if $c < 0$. Size of z-intercept increases quadratically in k. As $|k|$ increases, upward opening parabolas move downward, and vice versa.

$y = k:$ $\quad z = \frac{cx^2}{a^2} - \frac{ck^2}{b^2}$

Parabolas opening along the z-axis: upward if $c > 0$, downward if $c < 0$. Size of z-intercept increases quadratically in k. As $|k|$ increases, upward opening parabolas move downward, and vice versa.

$z = k:$ $\quad \frac{x^2}{\left(a\sqrt{|k/c|}\right)^2} - \frac{y^2}{\left(b\sqrt{|k/c|}\right)^2} = \pm 1$

Hyperbolas. Sign matches k/c. Open along x-axis if $k/c > 0$, y-axis if $k/c < 0$. Intercepts increase linearly in $|k|$. Collapse to a pair of lines through the origin when $k = 0$.