NUMBER THEORY
FALL 2020

ASSIGNMENT 1.2
DUE SEPTEMBER 2

Exercise 1. Prove that for all $n \in \mathbb{N}$, $15|2^{4n} - 1$. [*Suggestion:* Factor the polynomial $X^m - 1$.]

Exercise 2. Prove that the square of any integer has the form $3k$ or $3k + 1$. Use this to show that $3a^2 - 1$ is never a perfect square.

Exercise 3. If n is odd, prove that $16|n^4 + 4n^2 + 11$. [*Suggestion:* Use the fact that $11 = 16 - 5$.]

Exercise 4. Use the Euclidean algorithm to compute the following GCDs.

a. $(143, 277)$

b. $(306, 657)$

c. $(272, 1479)$