NUMBER THEORY
FALL 2020

ASSIGNMENT 2.2
DUE SEPTEMBER 9

Exercise 1. Let $a, b \in \mathbb{Z}$, not both 0. Prove that if $x, y \in \mathbb{Z}$ satisfy $ax + by = (a, b)$, then $(x, y) = 1$.

Exercise 2. Show that for any integer $a \in \mathbb{Z}$, $(2a + 1, 9a + 4) = 1$.

Exercise 3. If $a, b \in Z$ are not both zero, prove that $(2a - 3b, 4a - 5b)$ divides b. Conclude that $(2a + 3, 4a + 5) = 1$.

Exercise 4. If $a, b, n \in \mathbb{N}$, prove that $(a, b) = 1$ if and only if $(a^n, b^n) = 1$.

Exercise 5. Let $a, b, n \in \mathbb{N}$. Prove that $a|b$ if and only if $a^n|b^n$. [*Suggestion:* For the reverse implication, write $a = (a, b)r$ and $b = (a, b)s$ with $(r, s) = 1$. Apply the preceding exercise and conclude that $r = 1$.]