PUTNAM SEMINAR
FALL 2022

QUIZ 1
DUE AUGUST 29

Problem 1. Determine all possible values of the expression

$$A^3 + B^3 + C^3 - 3ABC$$

where A, B and C are nonnegative integers.

Problem 2. Let $S_1, S_2, \ldots, S_{2^n-1}$ be the nonempty subsets of $\{1, 2, \ldots, n\}$ in some order, and let M be the $(2^n - 1) \times (2^n - 1)$ matrix whose (i, j) entry is

$$m_{ij} = \begin{cases} 0 & \text{if } S_i \cap S_j = \varnothing; \\ 1 & \text{otherwise.} \end{cases}$$

Calculate the determinant of M.

Problem 3. Let h and k be positive integers. Prove that for every $\epsilon > 0$, there are positive integers m and n such that

$$\epsilon < |h\sqrt{m} - k\sqrt{n}| < 2\epsilon.$$

Problem 4. Prove that every nonzero coefficient of the Taylor series of

$$(1 - x + x^2)e^x$$

about $x = 0$ is a rational number whose numerator (in lowest terms) is either 1 or a prime number.