



PUTNAM SEMINAR  
FALL 2022

QUIZ 6  
DUE OCTOBER 10

**Problem 1.** Evaluate

$$\int_2^4 \frac{\sqrt{\ln(9-x)}}{\sqrt{\ln(9-x)} + \sqrt{\ln(x+3)}} dx.$$

**Problem 2.** For what pairs  $(a, b)$  of positive real numbers does the improper integral

$$\int_b^\infty \left( \sqrt{\sqrt{x+a} - \sqrt{x}} - \sqrt{\sqrt{x} - \sqrt{x-b}} \right) dx$$

converge?

**Problem 3.** Find the volume of the region of points  $(x, y, z)$  satisfying

$$(x^2 + y^2 + z^2 + 8)^2 \leq 36(x^2 + y^2).$$

**Problem 4.** Let  $p(x) = 2 + 4x + 3x^2 + 5x^3 + 3x^4 + 4x^5 + 2x^6$ . For  $0 < k < 5$  define

$$I_k = \int_0^\infty \frac{x^k}{p(x)} dx.$$

For which  $k$  is  $I_k$  smallest?