Number Theory
Fall 2023

Assignment 1.1
Due August 30

Exercise 1. Let $a, b, c \in \mathbb{Z}$ with $a \neq 0$. Use the fact that $\mathbb{Z}$ is a domain to prove that if $ab = ac$, then $b = c$. This is called the *cancellation law* in $\mathbb{Z}$. [*Suggestion.* If $ab = ac$, then $ab - ac = 0$. Now factor out a.]

Exercise 2. Let $a, b \in \mathbb{Z}$ with $b \neq 0$. Prove that if $b = ac$ for some $c \in \mathbb{Z}$, then c is unique. [*Suggestion.* Write $b = ac = ac'$ and use the preceding exercise.]

Exercise 3. Prove that divisibility in $\mathbb{Z}$ is transitive. That is, show that if $a, b, c \in \mathbb{Z}$ with $a|b$ and $b|c$, then $a|c$.

Exercise 4. Let $a, b \in \mathbb{Z}$. Show that if $ab = 1$, then $a, b \in \{\pm 1\}$ and $a = b$. [*Suggestion.* Argue by contradiction. Use the fact (proved in class) that if $a \neq 0$ and $|b| > 1$, then $|ab| > |a|$.]