NUMBER THEORY
FALL 2023

ASSIGNMENT 2.2
DUE SEPTEMBER 6

Exercise 1. For each of the following pairs of integers a and b use the Euclidean Algorithm and "q matrices" to compute (a, b) and express it as a linear combination of a and b.

a. $a = 24$, $b = 138$

b. $a = 119$, $b = 272$

c. $a = 1769$, $b = 2378$

Exercise 2. Recall that the *Fibonacci sequence* $\{f_n\}$ is defined by setting $f_0 = 0$, $f_1 = 1$ and $f_{n+2} = f_{n+1} + f_n$ for $n \geq 0$. What happens if you apply the Euclidean Algorithm to compute (f_n, f_{n+1})? More specifically, what is the GCD and what are the "q matrices"?

Exercise 3. Use the results of the preceding exercise to find $r, s \in \mathbb{Z}$ so that $rf_n + sf_{n+1} = (f_n, f_{n+1})$.