NUMBER THEORY
FALL 2023

ASSIGNMENT 4.1
DUE SEPTEMBER 20

Exercise 1. Let $n \in \mathbb{N}$. If $\sqrt{n} \in \mathbb{Q}$, then we can write $\sqrt{n} = \frac{a}{b}$ with $a, b \in \mathbb{N}$. Squaring both sides of this equation and clearing denominators yields $nb^2 = a^2$.

a. Let $\{p_1, p_2, \ldots, p_r\}$ be the set of prime factors of abn. Express a, b and n in modified canonical form using these primes and conclude that n is a perfect square.

b. Part **a** shows that if $\sqrt{n}$ is rational, then n is a perfect square. Formulate the (logically equivalent) contrapositive of this statement.

Exercise 2. Textbook exercise 3.1.15. Generalize "square" to "n^{th} power," for arbitrary $n \geq 2$.

Exercise 3. Textbook exercise 3.1.16. Be sure to also prove that the factors in the product of part (b) are unique.