NUMBER THEORY
FALL 2023

ASSIGNMENT 5.3
DUE SEPTEMBER 27

Recall that for $n \in \mathbb{N}$ we defined

$$\mathbb{Z}/n\mathbb{Z} = \{a + n\mathbb{Z} \mid a \in \mathbb{Z}\} = \{n\mathbb{Z}, 1 + n\mathbb{Z}, 2 + n\mathbb{Z}, \dots, (n-1) + n\mathbb{Z}\},$$

which is the set of congruence classes modulo n.

Exercise 1. Fix $n \in \mathbb{N}$ and let $a_1, a_2, \dots, a_n \in \mathbb{Z}$. Prove that the following are equivalent.

(i) The integers $a_1, a_2, \dots, a_n$ form a complete residue system modulo n.

(ii) For any $a \in \mathbb{Z}$ there exists an i so that $a_i \equiv a \pmod{n}$. [*Question.* How does this differ from the actual definition of "complete residue system modulo n?"]

(iii) For all $1 \le i, j \le n$:

$$i \neq j \quad \Rightarrow \quad a_i \not\equiv a_j \pmod{n}.$$

(iv) $\mathbb{Z}/n\mathbb{Z} = \{a_1 + n\mathbb{Z}, a_2 + n\mathbb{Z}, \dots, a_n + n\mathbb{Z}\}$ (in this case one says that $a_1, a_2, \dots, a_n$ *represent* $\mathbb{Z}/n\mathbb{Z}$).

Exercise 2. Textbook exercise 4.2.12.

Exercise 3. Textbook exercise 4.2.15.

Exercise 4. Textbook exercise 4.2.16.