Number Theory
Fall 2023

Assignment n
Ć. F. Gauss

Exercise 1. Prove that

$$1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$$

for all $n \geq 1$.

Solution. If n is even, then for $1 \leq k \leq n/2$ we may pair k with $n - k + 1$ in the sum in question, obtaining

$$1 + 2 + 3 + \cdots + n = \frac{n}{2}(k + (n - k + 1)) = \frac{n(n+1)}{2},$$

as claimed. If n is odd, we may still pair k with $n - k + 1$, but only for $1 \leq k < \frac{n+1}{2}$. This leaves the summand $\frac{n+1}{2}$ unpaired, so that in this case we have

$$1 + 2 + 3 + \cdots + n = \frac{n-1}{2}(k + (n - k + 1)) + \frac{n+1}{2} = \frac{n+1}{2}(n - 1 + 1) = \frac{n(n+1)}{2}$$

once again. □