



COMPLEX VARIABLES
FALL 2024

ASSIGNMENT 12.2
DUE DECEMBER 4

Exercise 1. Let $A \subset \mathbb{C}$ be a bounded domain whose (positively oriented) boundary ∂A is a finite union of piecewise smooth simple closed paths. Argue informally (without using the Cauchy Integral Formula) to explain why

$$\int_{\partial A} \frac{dz}{z - a} = 2\pi i$$

for any $a \in A$, even if A is *not* simply connected (i.e. A has “holes”). [*Remark.* One can make a formal argument using the Jordan Curve Theorem.]

Exercise 2. Textbook exercise 2.4.10. [*Remark.* You can also show that $F(z)$ is analytic using Morera’s Theorem, if you can prove that $F(z)$ is continuous. This isn’t difficult, but requires the use of *(local) uniform continuity* of $f(z, w)$.]

Exercise 3. Textbook exercise 2.4.13.

Exercise 4. Textbook exercise 2.4.14.

Exercise 5. Textbook exercise 2.4.18.