Exercise 1. Let P and Q be polynomials in z with complex coefficients.

a. Use Exercise 5.3.3 to show that

$$\lim_{z\to\infty} \frac{P(z)}{Q(z)} = \infty$$

if and only if $\deg P > \deg Q$.

b. Conclude that if P is nonconstant, then

$$\lim_{z\to\infty} P(z) = \infty.$$

Exercise 2. Let f and g be complex-valued and defined in a neighborhood of $\alpha \in \mathbb{C} \cup \{\infty\}$. Show that if $\lim_{z\to\alpha} f(z) = a \in \mathbb{C}^\times$ and $\lim_{z\to\alpha} g(z) = \infty$, then

$$\lim_{z\to\alpha} f(z)g(z) = \infty.$$

[*Suggestion.* Use Exercise 4.3.3.]

Exercise 3. If we represent $\mathbb{R}^3$ by $\mathbb{C} \times \mathbb{R}$, the standard equation for the unit sphere S^2 becomes $|z|^2 + v^2 = 1$, where $(z, v) \in \mathbb{C} \times \mathbb{R}$. The complex plane itself is represented by points of the form $(z, 0)$, or the equation $v = 0$. Let $N = (0, 1)$ represent the north pole of S^2

a. Define $\pi : S^2 \setminus \{N\} \to \mathbb{C}$ as follows. Given a point $p \in S^2 \setminus \{N\}$, draw the line through N and p, and define $\pi(p)$ to be the point where this line meets the complex plane $v = 0$. Show that if $p = (z, v) \in S^2$, then

$$\pi(z, v) = \frac{z}{1 - v}.$$

b. Conversely, use the same construction to show that

$$\pi^{-1}(z) = \left(\frac{2z}{|z|^2 + 1}, \frac{|z|^2 - 1}{|z|^2 + 1} \right)$$

for $z \in \mathbb{C}$.

c. Show that $\lim_{z\to\infty} \pi^{-1}(z) = N$.