Complex Variables — Assignment 7.2
Fall 2024 — Due October 16

Exercise 1. Let $\gamma : [a,b] \to \mathbb{C}$ be a path. We say γ is *differentiable* at $t_0 \in (a,b)$ if

$$\gamma'(t_0) = \lim_{t \to t_0} \frac{\gamma(t) - \gamma(t_0)}{t - t_0}$$

exists. If we write $\gamma(t) = x(t) + i\,y(t)$, it is easy to show that γ is differentiable at t_0 if and only if the (real-variable/real-valued) functions $x(t)$ and $y(t)$ are, and in this case

$$\gamma'(t) = x'(t) + iy'(t).$$

Use the multivariate chain rule (for real-valued functions) and the Cauchy-Riemann equations to prove that if $A \subseteq \mathbb{C}$ is open, $f : A \to \mathbb{C}$ is analytic on A, and $\gamma : [a,b] \to A$ is a differentiable path in A, then $f \circ \gamma : [a,b] \to \mathbb{C}$ is a path in the range of f, and the chain rule holds:

$$\frac{d}{dt} f(\gamma(t)) = f'(\gamma(t)) \cdot \gamma'(t) \quad \text{for all } t \in (a,b).$$

Exercise 2. Textbook exercise 1.5.17.

Exercise 3. Textbook exercise 1.5.20.