



COMPLEX VARIABLES
FALL 2024

ASSIGNMENT 9.1
DUE NOVEMBER 6

Exercise 1. Let $\emptyset \neq A \subset \mathbb{C}$ be open, let $f : A \rightarrow \mathbb{C}$ be continuous, and let $\gamma : [a, b] \rightarrow \mathbb{C}$ be a piecewise smooth path whose image lies in A . If $a = t_0 < t_1 < t_2 < \cdots < t_n = b$ and γ is smooth on each $[t_{j-1}, t_j]$, we defined

$$\int_{\gamma} f(z) dz = \int_a^b f(\gamma(t)) \gamma'(t) dt = \sum_{j=1}^n \int_{t_{j-1}}^{t_j} f(\gamma(t)) \gamma'(t) dt.$$

Prove that this is independent of the partition $P = \{t_j\}$ of γ into smooth pieces. [*Suggestion:* If P and P' are both partitions of $[a, b]$ on whose subintervals γ is smooth, show that the integral using either partition is equal to the integral using the common refinement $P'' = P \cup P'$.]

Exercise 2. Textbook exercise 2.1.1.

Exercise 3. Textbook exercise 2.1.7.